Onion ORAM: Constant Bandwidth ORAM with Server Computation

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Current art and where to go next

- **State of the art schemes**
  - Bandwidth: O(log N)
  - Client storage: O(1)   (Path ORAM = O(log N))
  - Server storage: O(N)

- **Is "optimal" ORAM possible?**

  O(1) bandwidth, O(1) client storage, O(N) server storage

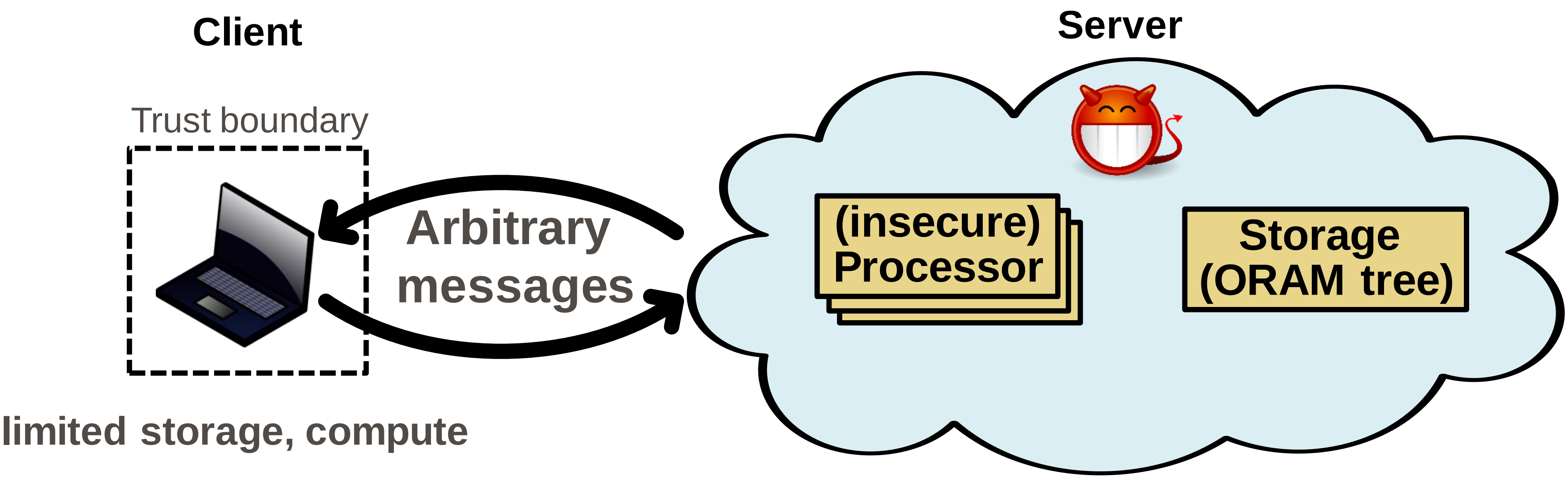
- **Goldreich-Ostrovsky lower bound [1987, 1996]**

  Given a program that runs in T time and an N block ORAM with O(1) client storage, the program+ORAM must run in $\Omega(T \log N)$ time

- **$\Omega(T \log N)$ doesn't mean $\Omega(T \log N)$ bandwidth!**

ORAM with Server Computation

- **Example: Outsourced storage (Honest but curious)**



- **“Read X, Y, Z, return F(X, Y, Z)”**
- **Message stream must be oblivious**

Server comp. in previous ORAMs

- **XORing reads [Dautrich et al.], PIR+ORAM [Mayberry et al.]**

- **XOR + Ring ORAM**
  - Permuted buckets → one real block touched / read
  - **B**, d1, d2, d3, …
  - **E(B, r)**, E(0, r1), E(0, r2), E(0, r3) …
  - **Server sends: E(B, r)** ⊕ E(0, r1) ⊕ E(0, r2) ⊕ E(0, r3) ⊕ ...
  - **Client computes:** E(0, r1) ⊕ E(0, r2) ⊕ E(0, r3) ⊕ ...

- **Both schemes make read bandwidth O(log N) → O(1)**
- **Does not help on evictions!**

Can we make evictions O(1) Bandwidth?

FHE + ORAM

- **Example: Ring ORAM**
  - ORAM on server is encrypted under FHE scheme E^{FHE}
  - Reads

Client **Server**

leaf, E^{FHE}(address) →

$E^{FHE}(d)$ = Select(E^{FHE}(address), Path(leaf))

RemoveBlock(E^{FHE}(address), Path(leaf))

← **$E^{FHE}(d)$**

  - Evictions

Path($leaf_g$)' = EvictPath(Path($leaf_g$))

Read bandwidth is O(1), no bandwidth for evictions!

Eviction circuit for FHE

$\text{Path}(\text{leaf}_g)' = \text{EvictPath}(\text{Path}(\text{leaf}_g))$

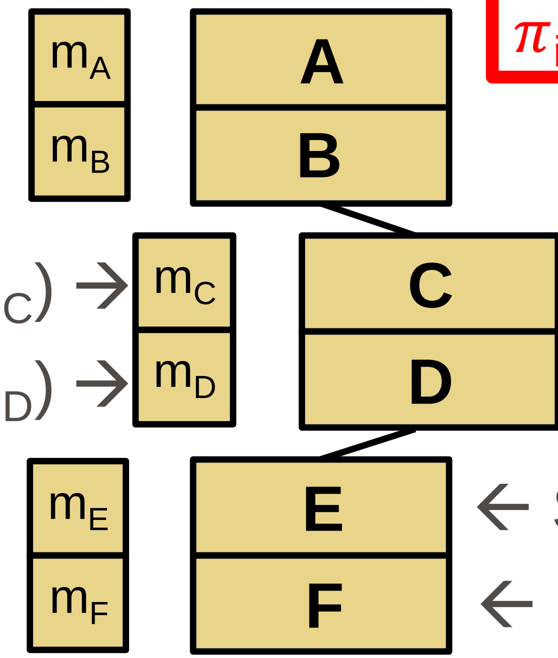
m_A = metadata for block A

$\pi_i = \text{MoveBlock}(m_A \ldots m_F)$

Block C can get stuck

$\text{Select}(\pi_C, m_A, m_B, m_C) \rightarrow$

$\text{Select}(\pi_D, m_A, m_B, m_D) \rightarrow$



$\leftarrow \text{Select}(\pi_C, A, B, \mathbf{C})$

$\leftarrow \text{Select}(\pi_D, A, B, \mathbf{D})$

$\leftarrow \text{Select}(\pi_E, A, B, C, D, \mathbf{E})$

$\leftarrow \text{Select}(\pi_F, A, B, C, D, \mathbf{F})$

- **Only Select() touches *blocks***
- **Server computation: polylog(N)**
- **Bootstrap to manage noise [Apon et al., Mayberry et al.]**

*Discuss later: Does the previous scheme achieve optimal Bandwidth/storage?

Do we need bootstrapping?
Do we need FHE?

Do we need FHE?

- **Additive-HE (e.g., Paillier)**
  - Addition: $E^{AHE}(a) \oplus E^{AHE}(b) = E^{AHE}(a+b)$
  - Scalar multiplication: $E^{AHE}(a) \otimes c = E^{AHE}(ca)$

- **Select from (X, Y):**

$$\mathbf{E^{AHE}(0) \otimes X \oplus E^{AHE}(1) \otimes Y = E^{AHE}(0+Y) = E^{AHE}(Y)}$$

- **Y = E^{AHE}(plaintext)**
- **Select op → $E^{AHE}(E^{AHE}$(plaintext))**
  - Client decrypts twice
  - (Possible) ciphertext blowup per layer

Block gets extra layer of encryption

- **Layers(output) = *max*(Layers(Block_i) : Blocks) + 1**

ORAM Read + Additive-HE

- **ORAM encrypted using 1 layer of E^{AHE} (abbreviated E)**

Client | **Server**

leaf →

← E(metadata for Path(leaf))

Decrypt metadata

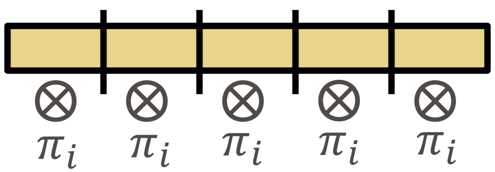
Compute $\pi = E(0), E(0), \ldots$ $E(1)$, $\ldots E(0)$

π, E(updated metadata for Path(leaf)) →

Write updated metadata

Compute E(E(d)) = Select(π, Path(leaf))

Block i



← E(E(d))

Decrypt d = E(E(d))

Update d → d'

E(d') →

Path(leaf)[root].append(E(d'))

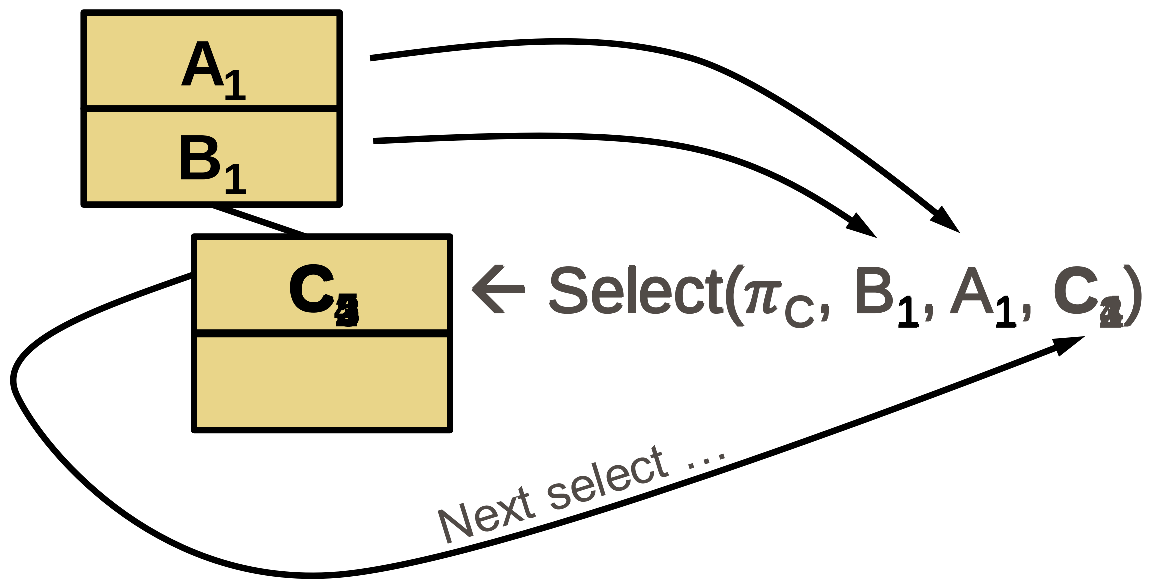
ORAM Evict + Additive-HE

- **Problem: Continuous reshuffling → Unbounded layers**

- **Reason: Blocks can get stuck in buckets after evictions**

Layers(output) = max(Layers($Block_i$) : Blocks) + 1

A_1 = Block A
with 1 layer



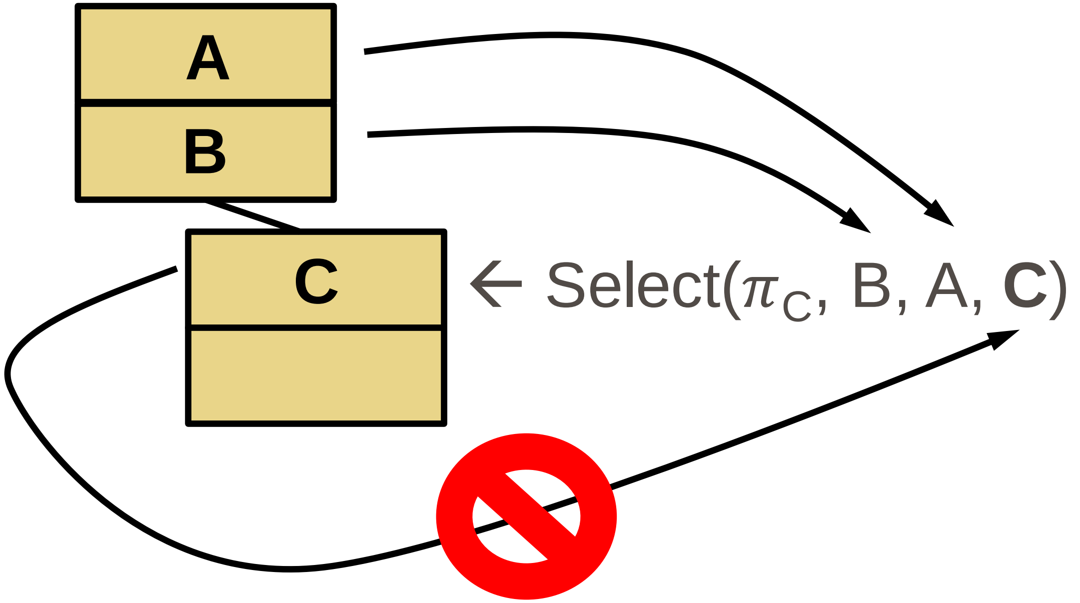
O(T) evictions → Slot with C gets O(T) layers

ORAM with O(1) bandwidth,
O(1) client storage,
O(N) server storage

…with only additive-HE

Design our ORAM eviction algorithm such that

buckets are guaranteed to be empty regularly



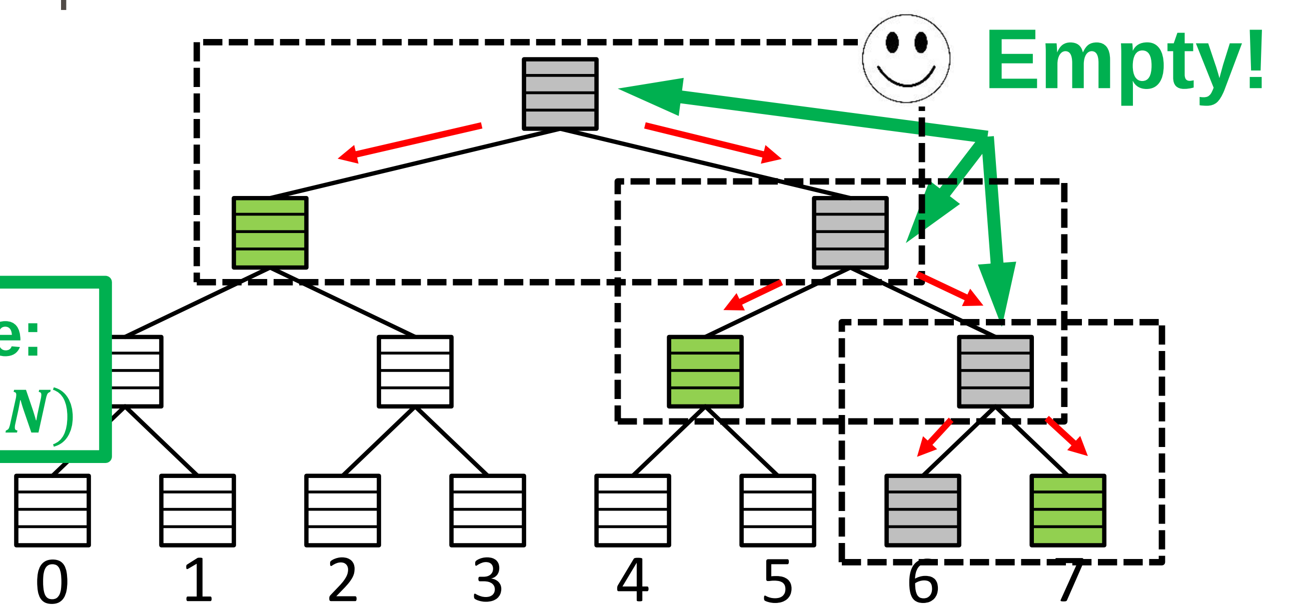
Onion ORAM

Design our ORAM eviction algorithm such that
buckets are guaranteed to be empty regularly

1. **Evict over reverse-lexicographic order of paths**
2. **Also evict to sibling buckets**
3. **Set Z, A s.t. Pr[bucket overflow] = negl(security parameter)**
4. Evict to 1 bucket triplet at a time

Example:
evict to leaf 6

Informal guarantee:
Max layers = $O(\log N)$



Which A, Z work?

Theorem: $Z \geq A$, $N \leq A * 2^{L-1}$

$\rightarrow$ **Pr[bucket overflow] =** $e^{-\frac{(2Z-A)^2}{6A}}$

- $Z = A = \theta(\log N)\omega(1) \rightarrow$ **Pr[bucket overflow] =** $N^{-\omega(1)}$

  $Note$: $N = \text{poly}(security\ parameter)$

- **Asymptotics w/o server computation**
  - Bandwidth = $O(\log^2 N)\omega(1)$ blocks
  - Client storage = $O(\log N)\omega(1)$ blocks
  - Server storage = $O(N)$ blocks

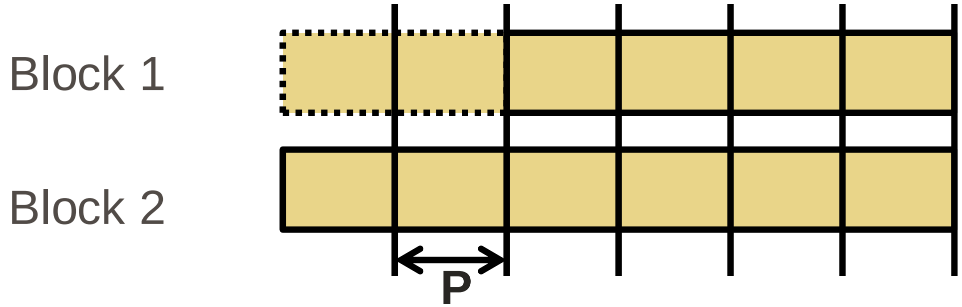
- **Not competitive w/o server computation**

Onion ORAM read w/ Additive-HE

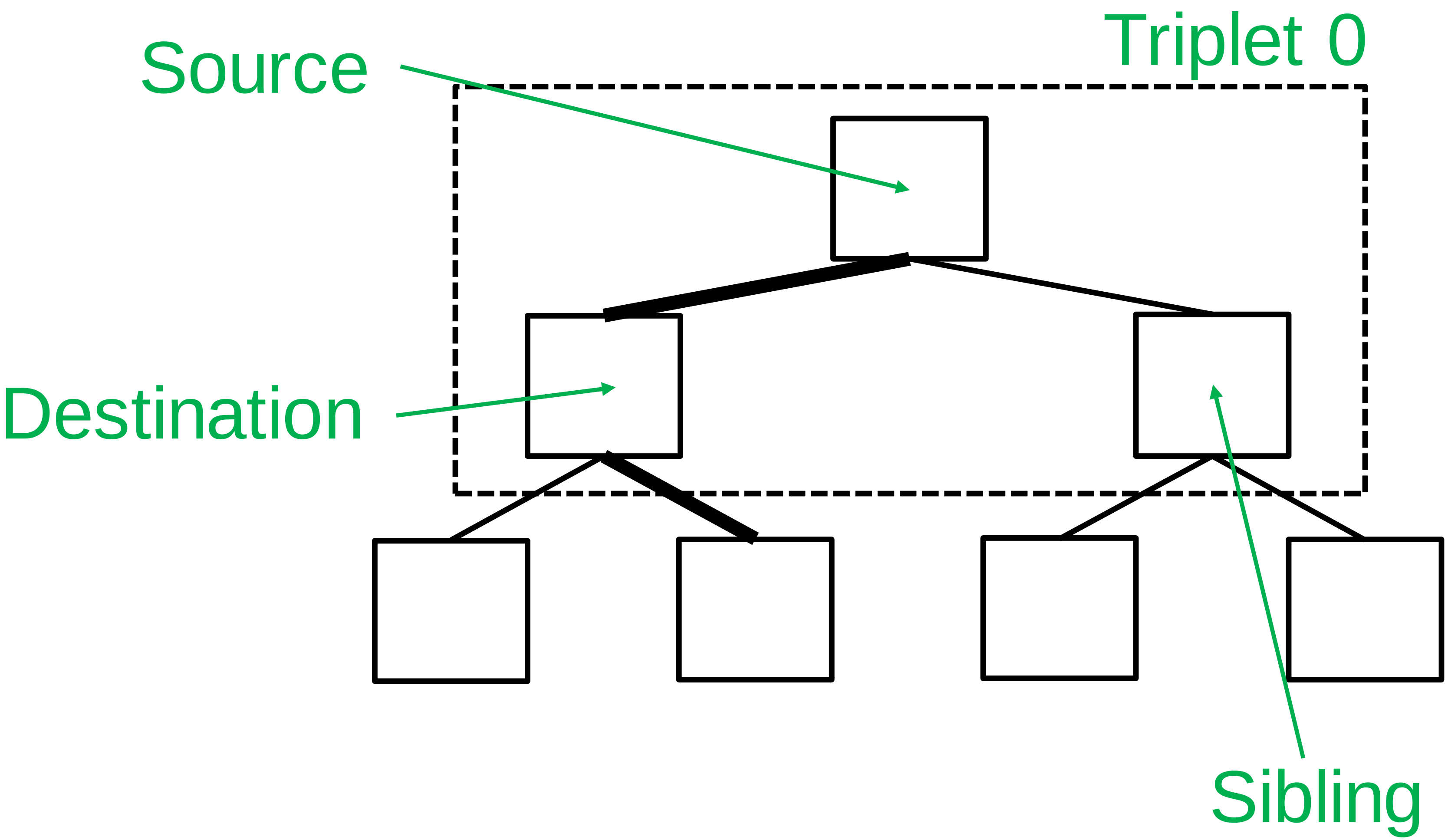
- **Same as previous proposal**
  - Client sends leaf
  - Server sends metadata
  - Client sends π = *E(0), E(0), … E(1), … E(0)*
  - Server sends block

Assume layers → ciphertext blowup

- **Simple scheme factoring in layers**
  - Elements of π have 1 layer
  - Pad blocks on path to S = Max($|Block_i|$: Blocks) bits
  - Split each padded block into C chunks s.t. S / C = Plaintext(π_i) = P



Eviction Terminology



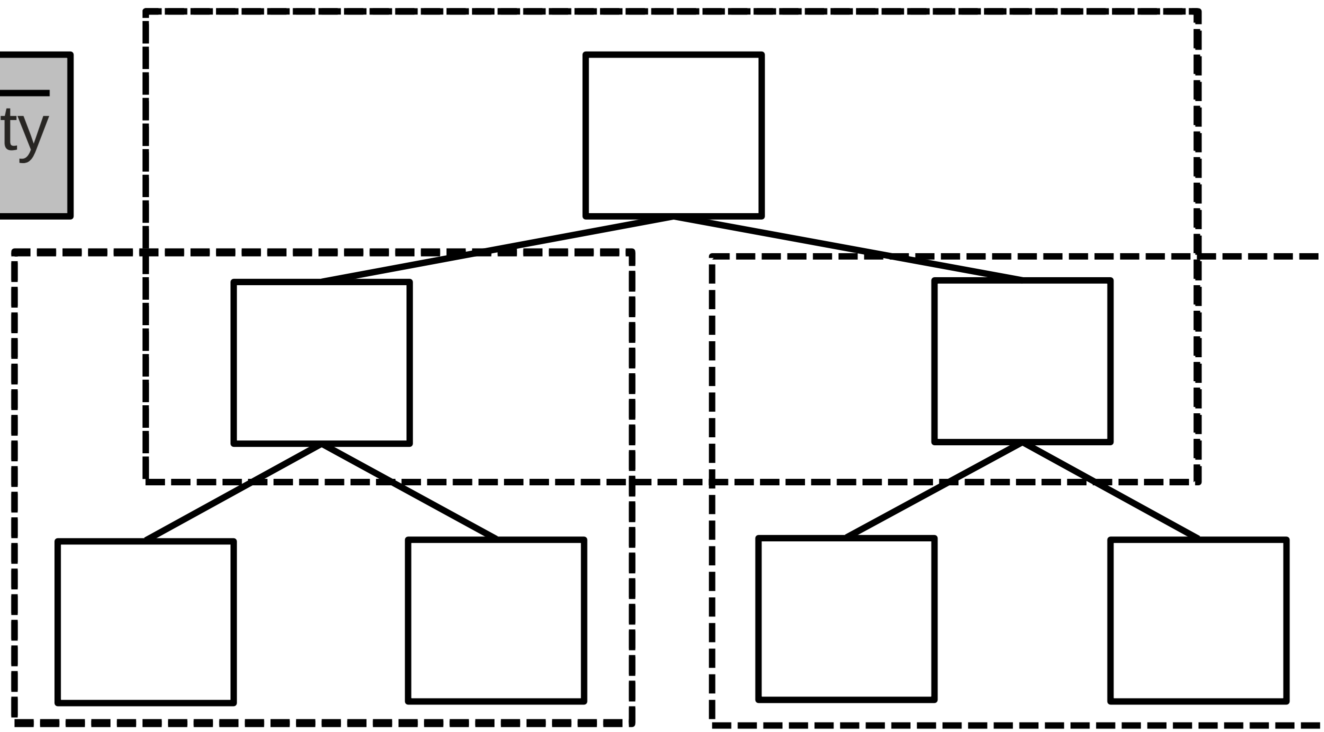
Path(leaf)[*i*] = i^{th} triplet on path

Path(leaf)[*i*].dest[*j*] = j^{th} block in i^{th} triplet's dest. bucket

Layer Analysis

- **Useful properties:**
    1. At eviction start: non-leaf sibling buckets are empty
    2. At eviction end: non-leaf destination buckets are empty

empty $\overline{\text{empty}}$



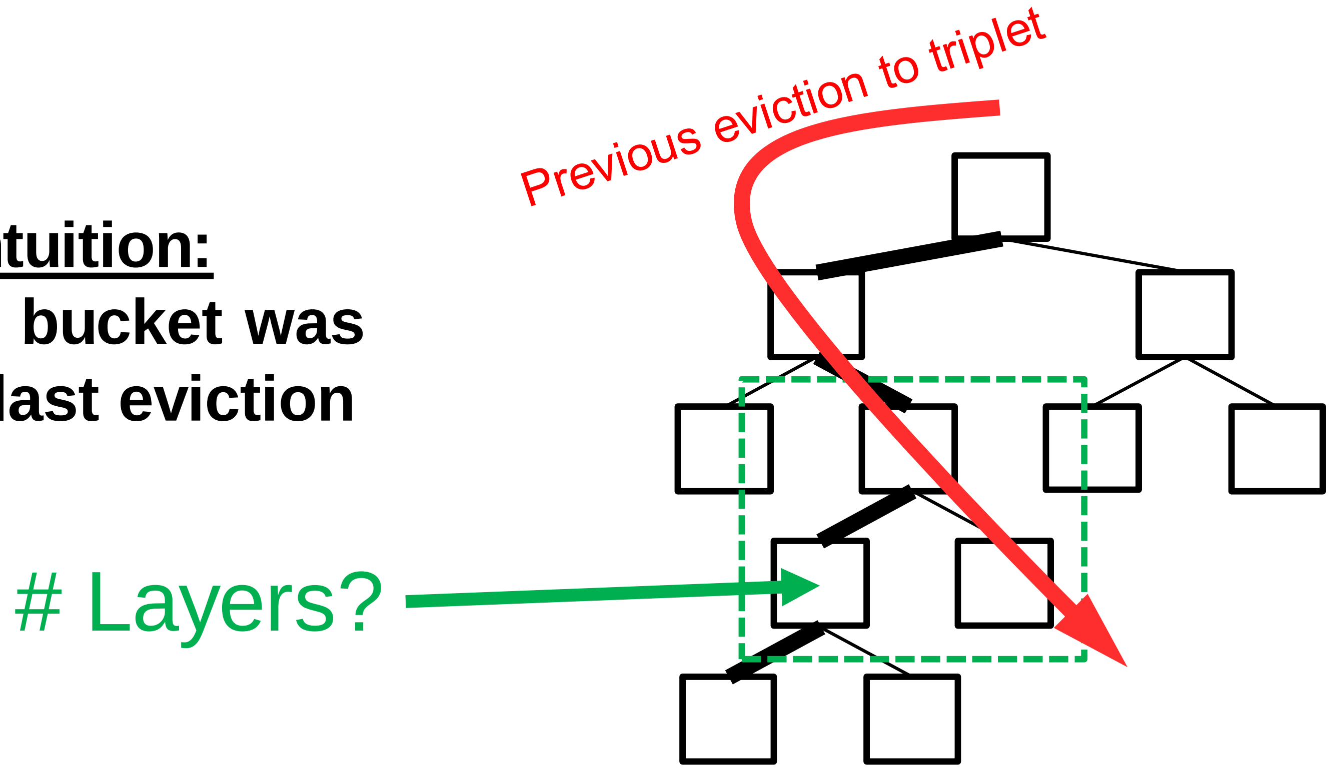
Leaves:

- Blocks get stuck in the leaves
- Non-leaves empty at regular intervals

Layer Analysis

- **Analyze: Layers on destination bucket at start of select**

<u>Key intuition:</u>
destination bucket was sibling on last eviction



Theorem: buckets at level $k < L$ have $\leq c * k + 1$ layers

- **c is constant, $c = 1$ in our final scheme**

Onion ORAM evict w/ Additive-HE

Client **Server**

leaf$_g$ (eviction path) known by server

E(metadata for Path(leaf$_g$))

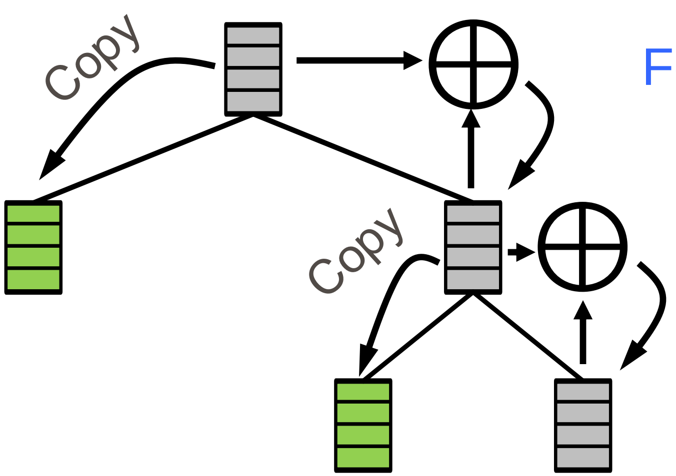
←

Compute $\Pi = \{\pi_0 \ldots \pi_{Z*L}\}$

($|\pi_i| = O(Z)$ encrypted coefficients)

Π, E(updated metadata for Path(leaf$_g$))

→



For triplet *i*:

Path(leaf$_g$)[i].sibling = Path(leaf$_g$)[i].src

For slot *j*:

args = {Path(leaf$_g$)[*i*].dst[j] , Path(leaf$_g$)[*i*].src}

Path(leaf$_g$)[i].dst[*j*] = Select(π_{Z*i+j}, args)

Eviction Post-Processing

Problem: layers in leaves are not bounded

- **At end of each eviction…**

Client **Server**

For j = 0 … Z - 1

$E(d_j) = Path(leaf_g)[L].dst[j]$ ←

Decrypt $E(d_j)$ to a constant number of layers, yielding e_j

Compute $E(e_j)$

$E(e_j)$ →

- **Layer theorem now applies to all levels**
- **Adds constant amortized bandwidth if Z ~ A**

Setting parameters

Which Additive-HE scheme?

Problem: each layer can add ciphertext blowup

- **Layer bound =** $\boldsymbol{O(\log N)}$
- **Paillier (1999):** $\boldsymbol{n \rightarrow n^2}$ **(n = RSA modulus)**
- **Damgård-Jurik (2001):** $\boldsymbol{n^s \rightarrow n^{s+1}}$
  - s = free parameter
  - Strategy: set $s_0 = \log N$, $\log N$ layers → $\boldsymbol{n}^{s_0 + \log N} = \boldsymbol{n}^{O(\log N)}$

- **Operations are like Paillier:**

$$E(a) \oplus E(b) = E(a)E(b) \qquad E(a) \otimes b = E(a)^b$$

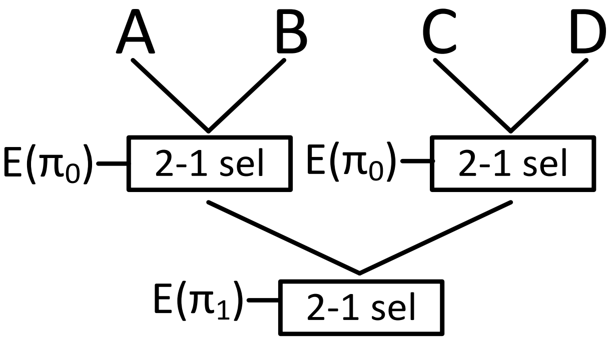
- **Best attack: factor** $\boldsymbol{n}$**, complexity** $\boldsymbol{\exp\left(|n|^{1/3}(\log |n|)^{2/3}\right)}$

$\therefore\ \boldsymbol{|n| = \theta\left(\log^3 N\right)}$ **→ defeat attacks w/ complexity** $\boldsymbol{N^{\omega(1)}}$

Optimization: Hierarchical PIR

- **So far … Select = $\bigoplus_i \pi_i \otimes Block_i$ "trivial linear PIR"**
  - Each select adds 1 layer
    $\text{layer bound} = \log N$
  - Z inputs → $|\pi| = Z * \text{layer bound} * |n| = log^5 N\omega(1)$

- **Hierarchical PIR [Lipmaa 2005]**
  - Multiplexer tree
  - Z inputs → $|\pi| = \log Z$ coefficients
    → select adds $\log Z$ layers
  - $\therefore \text{layer bound}' = \log N \log \log N$
  - Z inputs → $|\pi| = \log Z * \text{layer bound}' * |n|$
    $= \log^4 N \ \log^2 \log N$



At least better in theory

Parameterization

- **Strategy: set $|\boldsymbol{\Pi}| = |\{\boldsymbol{\pi_0} \ldots \boldsymbol{\pi_{Z*L}}\}| = \boldsymbol{O(B)}$**
- **I.e., $\boldsymbol{\Pi}$ contributes constant (amortized) bandwidth**

- **Let** $Z = \mathrm{A} = \log N\, \omega(1)$
- $|\pi_{read}| = |n| * \log^2 N \omega(1)$ **(n = RSA modulus)**
- $|\pi_{evict}| = |n| * \log N \log^2 \log N$ **(mux tree)**
- $|\boldsymbol{\Pi_{evict}}| = |n| * \log^2 N \log^2 \log N = \log^5 N \log^2 \log N$

- **Final asymptotics:**
  - Block size $\mathrm{B} = \Omega(\log^5 N \log^2 \log N)$
  - Bandwidth = $O(B)$
  - Client storage = $O(B)$
  - Server storage = $O(BN)$

Ongoing/Future work

- **Decrease block size** $B = \Omega(k * \log^2 N \ \log^2 \log N)$
  - Modern schemes w/o computation: $B = O(\log^2 N)$
- **How?**
  - Server computation is $O(\log^2 N)\omega(1)$ blocks - is $O(\log N)$ possible?
  - Is there a suitable additive-HE scheme with $k = o(\log^3 N)$?

- **Protect against malicious servers**
  - Server performs select incorrectly
- **Improve Garbled RAM schemes?**
  - Use ORAM as a blackbox
- **Parameterization for SWHE for Onion ORAM**
  - No bootstrapping needed