The Paper Tower;
Towering over the Heavens

Brought to you by: S | A / T | C

20th March 2006

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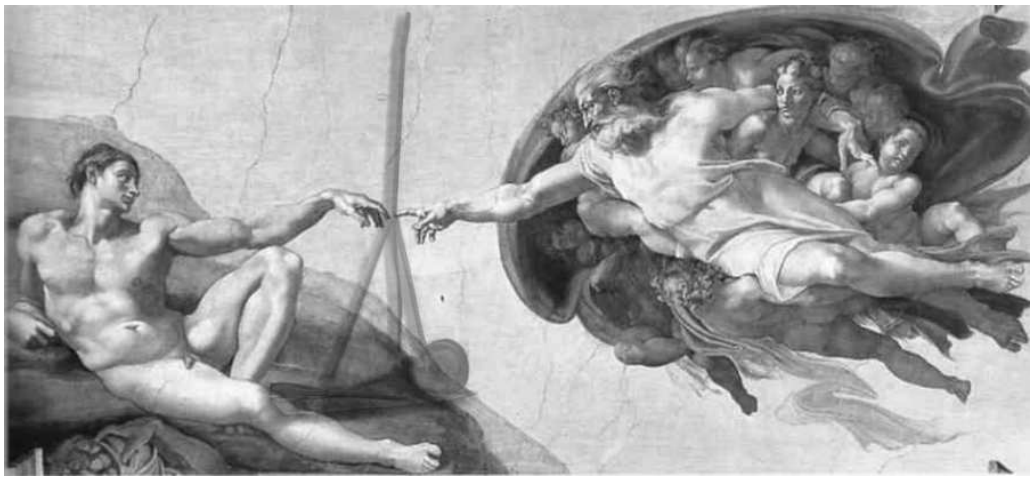

And God said, Let us make man in our image,
after our likeness: and let them have dominion over
the paper of the trees, and over the super strength 1.5 *cm*
scotch tape, fashioned from horse hoof over all the earth, and
over every ruler, and razor that creepeth upon the earth to
foster man's destructive tendencies.

So God created man in his own image, in the image of God
created he him; male and female created he them.

And God blessed them, and God said unto them,
go forth and build thy Lord a grand paper tower[1] *reaching*
upon the Heavens. Behold, I have bestowed upon you every
tool not native to this world, to wreak havoc down upon
it, and drive the innocent wilds from the forests, and to
construct grand paper towers from the trunks
that were their homes.

And to this man obeyed, and it was so, and
there was much rejoicing.

[1] In reality, the languages of the world that formed as a result of the tower of "Babel" were man's punishment for not using paper.

1 Introduction

The Paper Tower competition is Six Flags Magic Moutain's most rewarding competition. First place in this competition yields 12 free tickets to the park (4 for each of the 3 team members on the winning team). Second place yields 2 tickets for each team member.

Building a tall tower from one piece of paper sounds like a daunting task, but in reality, can be accomplished with a working strategy and muscle memory (practice). The current paper tower record is 273 cm (8'11.5"). With the strategy outlined in the following pages, a height of 258 cm (8'5.5") was achieved. Tallying height by slivers of paper cut lengthwise, this height differs from the record by $\frac{1}{2}$ **the length of a sliver**. In other words, with the design outlined in the following pages, you can beat the record down, scoring not only free tickets, but fame and an almost guaranteed A+ when you write this lab. Good luck!

2 Purpose

To build the tallest freestanding (for a minimum of 10 seconds) paper tower possible, with the allotted materials, in 45 minutes.

3 Materials

3.1 Materials that 6FM^2 provides

- Sheet of 8.5"x11" printer paper (1)
- Wooden 12" standard/metric (mm) ruler w/ a metal bar on side (1)
- Cardboard cutting board (1)
- Rectangular razor (1)
- Kindergarten (very bad and very small) scissors (1)
- 50 cm scotch tape (1)

3.2 Other materials that you are allowed to bring

- Mechanical pencil (3+)
- Stopwatch (1)
- Hand warmers/Heat packs (1+ box)

4 Theory

In theory, and as stated, all that three people need to make the tallest tower is a viable strategy and practice. Before the actual competition, team ASTC (**A**dam **S**owlati, **T**ristan Brown, **C**hristopher Fletcher) built 4 towers, achieving a personal best on the third try (not on Physics Day). Practice ensures that all 3 team members know exactly what to do, when to do it, and how to do it carefully. It most definitely goes without saying: practice is an absolute essential part of making any progress in the paper tower competition.

The second all important aspect to paper tower success is a working strategy. The tallest paper towers all use a very similar strategy. This details a tripod base supporting a spire of folded paper prisms (the ASTC specifications are shown in figure 1). Considering the fact that each prism is just under 1' (27.94 cm), towers shoot up very fast if constructed correctly. Too fast in some cases, this leading to teams not making sure that each addition still allows the tower to stand on its own. Ponder the problem: You and your group must ask yourselves, do you want to go for the record, or do you want to win? Trying to break the record increases the chance that your tower will not survive and might cost you the win. What is more important to you: free tickets in your pocket or fame eternal?

On Physics Day

When team ASTC practiced building the paper tower, we made the mistake of not taking into account what the actual circumstances of the competition would entail. First, during the competition, teams were only allowed $6FM^2$ tools (with the exception of pencils and a stopwatch). Don't bother practicing with a large pair of scissors; you won't be able to use them on the day of (the scissors they provide rival those of the old kindergarten scissors). From this information, learn to love the razor. Second, Valencia is a very cold place. When waiting in line to start the competition, which you will

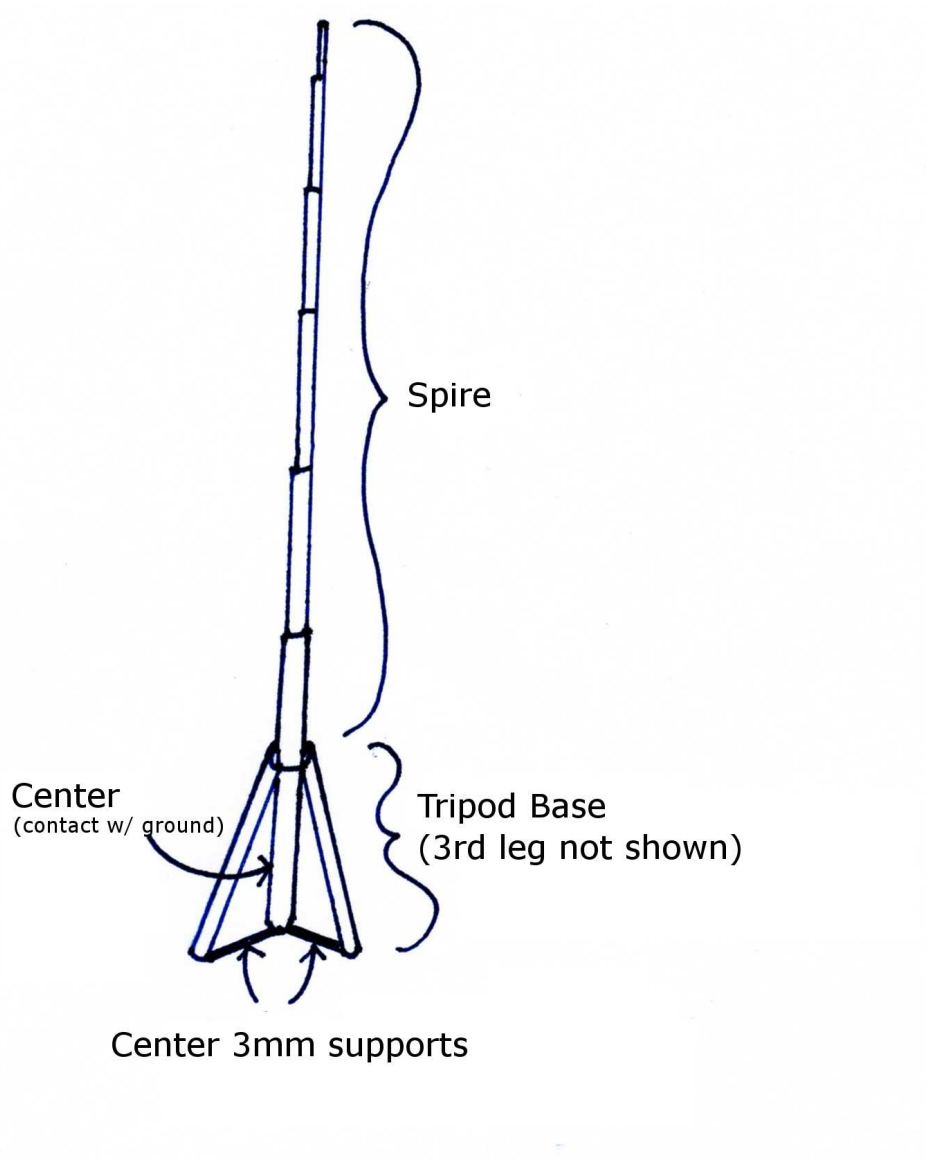

Figure 1: The ASTC Paper Tower.

need to do for at least 30 minutes in order to ensure that you have one of the 30 spots on the paper tower roster, your hands will freeze. This makes every task, save cutting with a razor, very tedious business. To this end, either bring a box of hand warmers/heat packs, or build your practice towers in the ice box. Anticipating your conditions during a competition is a key step to success, making 50 cm of difference in our case.

On Physics Day, beyond preparation, there are several environmental factors that can be taken advantage of to bolster success. First, as soon as the doors open (± 3 minutes before the competition begins), make a B-line to a corner of the room. Corner tables allow for towers to be leaned up against the corner wall while under construction. This becomes important in saving time because as the tower grows in height, one team member will have to work fulltime in standing the tower up[2]. Second, when the time comes to call a judge over to your table to measure height (each team can only get 2 recorded measures for height), wait for an older judge to walk by. If your tower is close to 8', the judge will have to stand on the table (with the tower on the floor) in order to measure accurately. This is very difficult, not to mention annoying, for older judges to do. Keeping in mind the fact that once you call a judge over, the judge won't leave until either a.) you tell the judge to go away, or b.) you manage to get your tower to stand for 10 seconds, you can really irritate your judge in the time that it takes to get a measurement. So follow the facts of life relationship:

$$Age^2 ToT \propto Leniency$$

where Age is your judge's age, ToT is time on table, and $Leniency$ is the reduction in the 10 second standing rule that the judge applies when grading the tower.

What? You want more advice? Or maybe just a laugh? Read on: Confucius say someone in your group should be able to solve the Rubix Cube in under 30 seconds. What does the Rubix Cube have to do with building paper towers, you ask? Intimidation. All the while that you're sitting in line, waiting for the event to start, you should have someone solving the Rubix Cube very impressively and very visibly, tearing down your opponents' nerves.

5 Procedure

5.1 Cutting

Cutting is the first order of business in building a paper tower. All cuts are made along the paper's length (the 11" side) and measured along the paper's width (the 8.5" side). All cutting is made by a single team member: the

[2] If, during the competition, the paper tower leans all the way over, game over. The crease created in the spire, from the fall, will have created irreversible structural damage that is very hard to overcome when supporting 8+ feet of tower.

cutter. The cutter's objective is to cut straight paper pieces that are of a set width for tripod pieces and of a steadily decreasing width for spire pieces. A decreasing width for spire pieces will create a pyramid effect: more support for the higher pieces and less weight for the lower pieces to support.

One of the team's most pressing challenges is to determine the decreasing width trend for spire pieces. The piece of paper given at the competition is 216 cm wide. Realistically, at least 93 cm will be donated to the tripod base. This leaves 123 cm to build the tower spire. Each piece cut lengthwise can yield a possible 27.94 cm in height. Because of paper constraints, it is absolutely essential to maximize the number of spire pieces. At the same time though, it is important not to make each piece too thin in order to yield more pieces. In making a tower 8'5.5", team ASTC used the following reducing width scheme:

Base (mm)
$24 \rightarrow (0 - 24)$
$20 \rightarrow (24 - 44)$
$20 \rightarrow (44 - 64)$
$20 \rightarrow (64 - 84)$
$9 \rightarrow (84 - 93)$

Spire (mm)
$22 \rightarrow (93 - 115)$
$20 \rightarrow (115 - 135)$
$18 \rightarrow (135 - 153)$
$15 \rightarrow (153 - 168)$
$13 \rightarrow (168 - 181)$
$11 \rightarrow (181 - 192)$
$10 \rightarrow (192 - 202)$
$8 \rightarrow (202 - 210)$
$6 \rightarrow (210 - 216)$

This plan of action can yield a maximum of 279.4 cm in height (counting the 24 mm piece in the base section). Don't count on a full 279.4 cm, though, as some height will be lost due to overlaps of pieces.

After determining a reducing width scheme, the first step towards actually cutting is measuring. All measurements should be marked by pencil along the paper's width. Measurements for each piece should be marked on both

sides of the paper. **After measuring each piece, label it on both sides (front and back of the paper) with the appropriate width value. This will ensure that no piece goes missing and that wider pieces do not go on top of narrower ones.** A common strategy is to mark each fold line as well as each cut line. A fold line is any point that each piece is to be folded (as will be discussed, each piece will be folded either once or twice). Team ASTC warns strongly against this. It takes too much time and does not supplement a good eye for folding. Additionally, unless the fold marks are perfectly on point with a very sharp pencil, the pencil mark width will create a source of error that can lead to folding slightly off the correct line.

The next and final step in the cutting phase is the actual cutting. When all cut marks are registered on both sides along the width of the paper, place the paper on the cutting board and unsheathe the razor. Line the metal side of the wooden ruler along the first (the 24 mm) notch and slide the razor down the metal to make the cut. While cutting, make sure that the razor is perpendicular to the paper. The metal side of the ruler is lifted slightly off the ground. If the cutter is not careful, the cut will be underneath the metal strip, which could create an uneven cut if rectified halfway down the paper. Continue this procedure over and over again until all pieces are cut.

When cutting, keep in mind that the other two team members depend on the paper pieces to do their work. To this end, it is essential to cut the pieces in the order that their widths are listed (above). The base should be cut out first (starting with the 24 mm piece). As pieces are produced, the other team members should go to work on folding and positioning them in the base (explained in the "Time Management" section). **Once the cutter is done cutting, he/she should help in the task of folding, taping, and building the spire.**

Special Piece Assignments

1. Once all pieces are cut out, the 9 mm piece (from the base section) should be sliced into three 3 mm wide pieces. These pieces become the center supports and hold the center piece in position relative to the tripod legs.

5.2 Folding

Folding is the most important and vital part of the paper tower contest. Why is it so important? Folding determines the shape of each segment of

the paper tower. When team ASTC created its first paper tower, and was ignorant of the best shape and folding techniques, it created a highly unstable structure. Likewise, using a cylinder shape wastes paper and thus restricts the maximum height of the paper tower. Thus, it is paramount that a proper shape and folding technique is found.

Team ASTC ultimately decided that an equilateral triangular prism would be the most stable structure (shown in figure 2). Why is this structure stable? First, a triangle is mathematically a strong and resistant structure. Indeed, many skyscrapers have been built using triangular shaped beams. Second, the equilateral triangle makes connecting the pieces of the paper tower extremely easy, as will be shown later. Third, this shape requires the *least amount of paper*. Fourth, the equilateral structure ensures that weight is balanced.

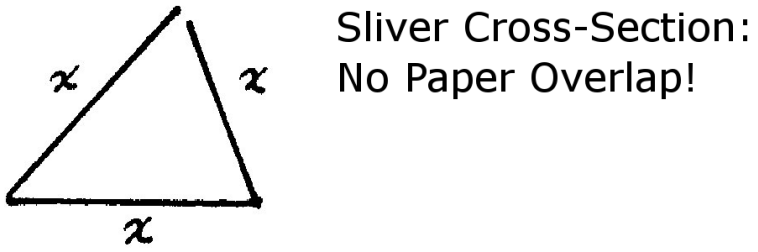

Figure 2: The equilateral triangle fold scheme.

A caveat, folding should be a one person job in order to create consistency among the pieces. When the cutter finishes his/her cutting, he/she should assist in folding iff the folder is falling behind. Further, and in general, all folders should be people with a steady hand and decent-to-great hand eye coordination.

After determining folding scheme and assignments, the task at hand is to learn to fold. As stated previously, the cutter should mark ever piece with its appropriate width in order to ensure that pieces aren't confused. As the cutter cuts each piece, the folder should be ready to start folding it. Time must not be wasted from the cutting board to the folding assembly line. Folding can be accomplished with rulers to ensure accuracy, however, team ASTC found that this is extremely and critically time consuming. In addition, it cannot be done until the cutter is finished with his/her ruler

because each team only gets one ruler. To compensate, the folder must visually observe where folds should take place (shown in figure 3). First, the folder should fold one third of the piece to the halfway point of the piece. This will create a fold that should subsequently be sharply creased with the fingers. Next, the other third of the paper should be folded over this flap. This second fold will completely cover the previous fold. At this point, the folder should run his/her fingers over the length of the piece to ensure that the piece has extremely sharp creases. Keep in mind, it is extremely important that the folds are the same width to ensure that the piece will form an equilateral triangular prism. In addition, it is paramount that the folder does not inadvertently bend the piece when creasing. Bending the piece horizontally will cause a great deal structural instability.

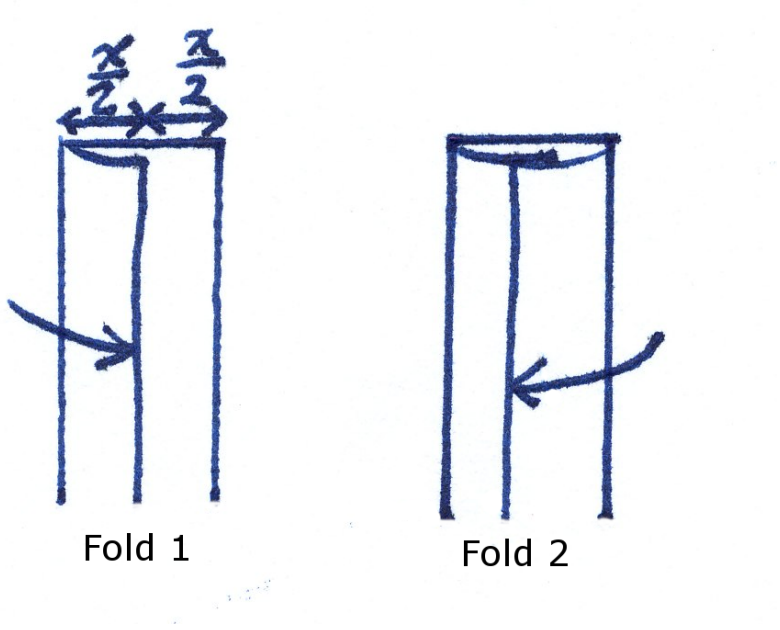

Figure 3: Folding a piece into a triangular prism.

Now the folder must open up each piece and thus create an equilateral triangular prism. Opening it up with hands is inefficient and could bend the piece. Instead, take a sharp pencil and run it through the length of the piece, starting at the opening at the end of the piece, while pulling the pencil up. This will have the effect of opening the piece and creating an equilateral triangular prism.

Once the folder has folded a piece and opened it up, it should be put aside in order of decreasing width. This will increase efficiency and reduce the time it takes to join the pieces together. **When all the pieces are folded and opened, the folder should proceed to taping and building the spire.**

Special Piece Assignments

1. The last 6 mm piece (from the spire section) should be folded in half only as it will be the final piece of the tower.

5.3 Taping

The job of taping belongs to every group member. Anyone who has run out of things to do should be grabbing folded strips and taping them into the beautiful triangular prisms that are so crucial to construction. Following the ASTC time management plan, the cutter and the folder will be the only ones not working on actual construction for the entire 45 minutes. This fact forces these two to be well-trained in the art of taping. The constructor will mostly just have to be good at taping different prisms together. **All three members should have very clean hands. Any oils from greasy foods or touching one's face will severely hinder the stickiness of the tape.**

After the taping job is understood, the next step is to tape. First, open the folded strips of a paper piece with a pencil slid under the flaps while the strip is pressed against the table, not your finger (if this has not been done already by the folder). Opening the strips with a finger will cause the eventual prisms to be curved. For creating the prisms, other guides advise taping lengthwise along the opening of the prism. Those guides are ***wrong***. Lengthwise taping may conserve tape, in that you're sealing more openings with less tape, but tape consumption really isn't an issue. Also, just trying to get a lengthwise piece to stick requires far too much time, effort, and pinching of the tower in ways that may destabilize it. Instead, tape with the length of the tape piece across the opening (shown in figure 4), and make sure that the taped point is well secured. The prisms should only require three pieces of tape (fewer higher up): one in the middle, and one 3 cm from each end.

5.4 Building the Base

As the first pieces cut are those of the base, the base builder should be ready to start on what is availible as quickly as possible. The base consists of the center (24 mm), 3 tripod legs (20 mm), and the center supports (9 mm

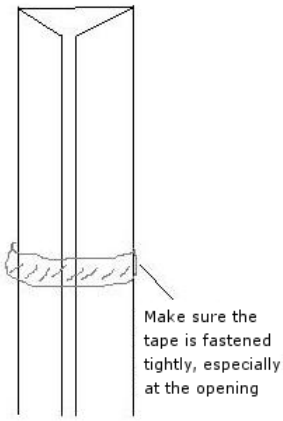

Figure 4: Taping a prism.

piece cut into 3 mm mini-pieces). Taping the tripod legs and center together is an art (shown in figure 5). To the base builder: start by attaching the flaps

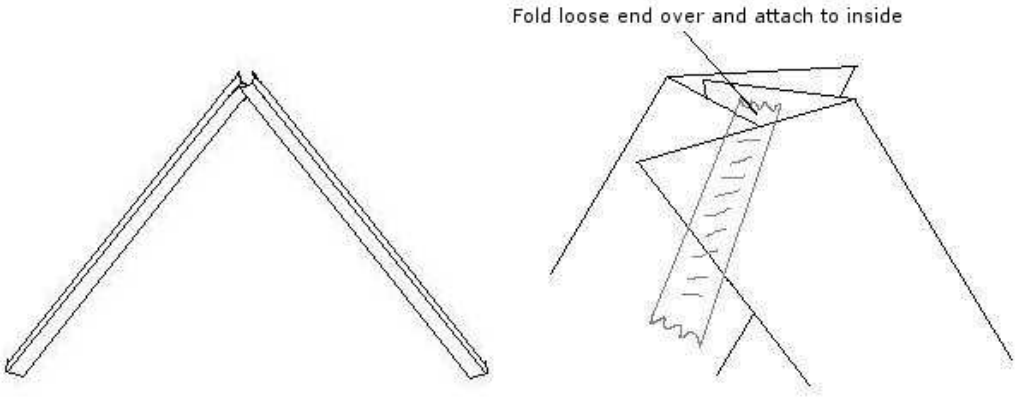

Figure 5: Taping the tripod legs together.

of two tripod legs together at about a $70 - 80^o$ angle. Before taping, hold the two legs together where the tape would go, and put the third leg where it will go to get a rough idea of how to make the base symmetrical. When you've gotten a good idea of the angle that you're going to use, attach the first two legs with only a thin strip of tape that will wrap around from the angle to the top. Any slight error in the angle of these legs can be corrected

in the way that the third leg is attached. Holding the third leg in place so that the base stands the way you want it, make sure once again that all the angles look symmetrical and the base looks high enough. Tape the two flaps of this third leg in the same way, making minor adjustments to location of the attachment as necessary to improve symmetry.

Once the base is free standing, it's time to use the scissors to trim the tripod. Cut slits about 4 mm long along the crease of each end of each leg that makes up the tripod (shown in figure 6). The slits at the apex will allow

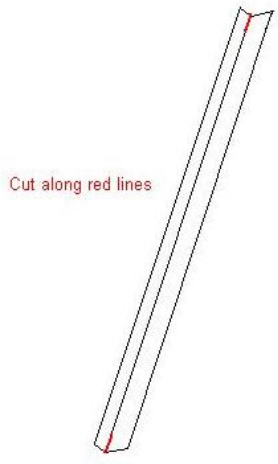

Figure 6: Cutting slits along the tripod legs.

you to fit the tower center (24 mm) into the tripod. The slits at the ends of the legs will allow you to add 3 stabilizing center support pieces (3 mm each) to the bottom. Tape up the widest supporting piece into a triangular prism. If the cross section of the prism isn't perfectly equilateral, **yell at your teammates to fold better**, because isosceles or scalene triangles can become weak points in pieces that aren't reinforced by the tripod[3]. Insert the center piece (24 mm) into the triangular hole at the apex of the tripod, and thread it through just about all the way to the ground.

Now that the center is secured, it is time to attach the 3 mm support pieces. Put a strip of tape on one end of one piece and attach it to one side of the center piece, perpendicular to that piece, near the ground. Put the other end of the piece through the slit in the nearest leg and, holding

[3]The base builder is the team's resident @#$ Hole and should be rocking the boat throughout the 45 minute journey to keep quality up.

the piece taught and the center piece perpendicular to the ground, tape the piece to the leg outside the slit (shown in figure 7). Do the same for the

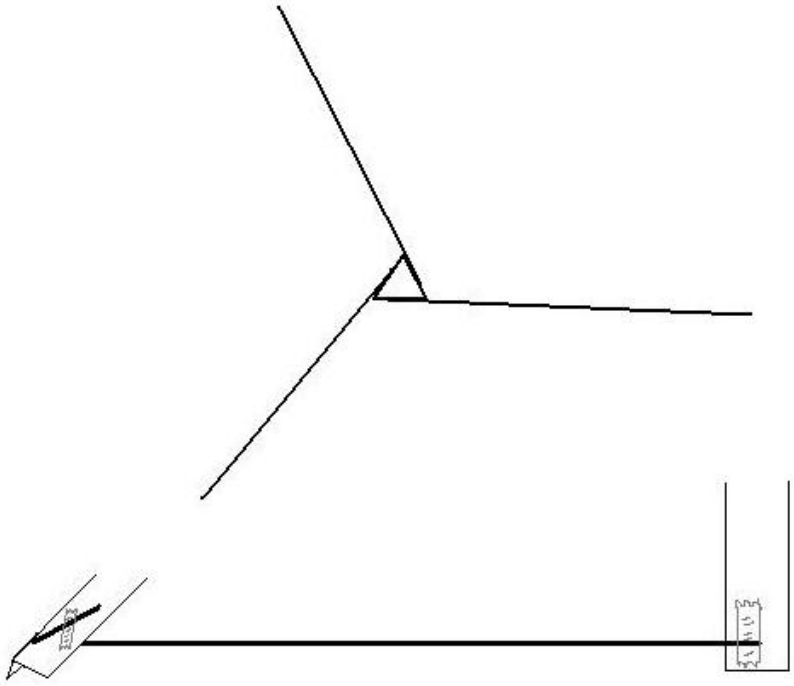

Figure 7: Connecting the center support pieces (3 mm) from leg to center.

other two pieces with the other two legs and the other two sides of the center piece. These pieces will allow you to adjust the tilt of the tower to aid in stabilization. Once you've got the center piece all straightened out, put a piece of tape or two at the apex of the tripod, connecting it to the center piece. Congratulations! The foundation is done. So is about half of 45 minutes, so if the other teammates aren't taping triangular prisms yet, then they need a scolding and a miracle.

5.5 Building the Spire

Building the spire is a 3 person job that starts ideally at the 20 minute mark with the base builder having completed his/her task. When the other two team members are done with folding and taping, they should begin

attaching pieces together in groups of two[4]. Because the pieces have been cut to be decreasing in width as you move up the tower, the constructors must be conscious of the width difference both in choosing the next piece to attach and in how these pieces are taped together.

The first order of business after the base is complete is to fit the second largest prism (22 mm) into the center prism (24 mm), if it's folded properly. If the folder and taper still aren't done folding equilateral prisms at this point, a significant amount of time could potentially be wasted while waiting for them.

When fitting prisms together, **always rotate the next prism so that the open side of the prism isn't lined up with the open side of the previous one.** Also, overlapping is important. Towards the bottom, fit the next prism about 2 cm into the previous prism. This amount can gradually become less as you move up the tower. Once you've fit the prisms together, tape them in a way that will stabilize the tower, or in other words, try to counterbalance any tilt of the tower. If a large portion of the tower is tilting one way, it might be better to readjust the support pieces (3 mm) at the bottom rather than try to correct the error higher up.

The constructor should be someone with a fine touch and the ability to balance even the most irregular objects. Despite all efforts and countermeasures to keep the tower straight, it will inevitably have kinks and a sway. The best way to minimize this is use the last of your tape. Locate the weak points by letting it tilt a bit while a spotter watches it. Tape points as well as any prisms that seem too open. Adjust weak or tilting joints with more tape. No matter how many times this is done, there will be imperfections, but the more the team practices, the better the tower will be. Keep in mind, once all group members come together to finish the spire, it is time to expend resources. Use remaining tape. One piece of tape administered in the right location can be the difference between a standing/unstable tower.

6 Time Management

Even after every task is understood, time management is still a crucial part in building a tower in 45 minutes. Every tower team has 3 team members.

[4]NOTE: Leave the highest two pieces single, so that if your tower can't support both last pieces, you can at least try to get one up.

That means that the paper tower competition is composed of $45 \cdot 3 = 135$ man minutes of work. To best put this time to use, team ASTC recommends the following time/work allocations for each team member:

Team Member #1: TM1 is the primary cutter (callsign "Cutter"), auxiliary taper, and auxiliary spire builder.

Time (*min*)	Cutting	Folding	Taping	Base Constr.	Spire Constr.
0-5	√				
5-10	√				
10-15		√			
15-20			√		
20-25			√		
25-30					√
30-35					√
35-40					√
40-45					√

Team Member #2: TM2 is the primary folder (callsign: "Folder"), auxiliary taper, and auxiliary spire builder.

Time (*min*)	Cutting	Folding	Taping	Base Constr.	Spire Constr.
0-5		√			
5-10		√			
10-15		√			
15-20		√			
20-25			√		
25-30					√
30-35					√
35-40					√
40-45					√

Team Member #3: TM3 is the base builder (callsign: "Slave Driver, Base Builder, Constructor") and the primary spire builder.

Time (*min*)	Cutting	Folding	Taping	Base Constr.	Spire Constr.
0-5				√	
5-10				√	
10-15				√	
15-20				√	
20-25					√
25-30					√
30-35					√
35-40					√
40-45					√

7 Endnotes

Start practicing at least a week ahead of time, and continue practicing until it's time to perform. Team ASTC's last practice session yielded a 8'5" tower that could stand for more than 10 seconds. Only through practice did team ASTC sweep with first place. Practice building in low light. Practice cutting with a dull razor. Practice racing against the clock so that you have more time to correct for error. The more you do it the better you'll get.

On a less serious note, though, there is still one environment-type advantage that we have not yet disclosed. While it may not be true for every grade, our grade has a perfectly expendable student enrolled in physics. If your grade has the same (or similar) read on:

> **Confucius' Vault**
> On Physics Day, the expendable student should enter the park alone and under the pretense that he/she is there for the rides. He/she should have smuggled a meter stick (preferably metal) into the park. Once inside, direct the rogue expendable student to the paper tower competition restaurant (it realty is a restaurant, or was during ASTC's appearance). Once the competition begins, the rogue student should start pacing around the room looking for the tallest towers. At the 20 minute mark (after the towers

begin to spring up, but before they can get measured), the rogue student should select the 3 highest enemy towers, take out his/her ruler and go on a rampage. What happens to the student after the top towers is not important, as long as he doesn't give away his identity.